INSTABILITY IN A SELF-MODELING CRYSTALLIZATION FRONT IN A ONE-COMPONENT MELT

V. V. Mansurov and N. G. Fomina

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Studies have been made on the stability of planar, cylindrical, and spherical crystallization fronts in a one-compound melt in the presence of small perturbations in the phase-transition surface.

There are several papers [1-5] on the morphological stability of a phase-transition front. These studies are important because, e.g., the loss of stability in a planar crystallization front results in a dendritic structure, which has a marked effect on the properties of the metal.

Here we continue a study of the stability in self-modeling processes begun in [1], in which crystallization is restricted by a single diffusion process occurring under isothermal conditions. Here we consider thermal conductivity without allowance for diffusion, and we examine the temperature perturbations in the crystal and in the melt. Methods from the theory of hydrodynamic stability are used to examine the stability of the crystallization front in a one-component melt with respect to small nonstationary perturbations in front shape. The results confirm the conclusions of [1] and eliminate the inaccuracies in [2-4], which arose because the small perturbations imposed on the crystallization front were stationary. It is shown that if the crystallization front is a planar, cylindrical, or spherical surface and if the melt is supercooled, then such a front is unstable. This has been observed by experiment [9] for lead, tin, and zinc. It occurs physically because the supercooling increases away from the crystallization front and any part of the surface of the crystal that finds itself in a region of higher supercooling grows more rapidly than the planar front.

In [5] it was concluded that the perturbations should increase in proportion to $\exp(\alpha t^{1/4})$; here we analyze this conclusion and show that it cannot be taken as reliable. The perturbations should increase in accordance with the exponential law $\exp(\beta t)$.

We consider the stability of a planar surface in the crystallization of a one-component melt. The unperturbed process is described by the solution to

$$a_i \frac{\partial^2 T_i}{\partial x^2} = \frac{\partial T_i}{\partial t}, \quad i = 1, 2, \tag{1}$$

$$T_1 \to T', \ x \to +\infty, \ T_2 \to T'', \ x \to -\infty, \tag{2}$$

$$\frac{dX}{dt} = -\lambda_1 \frac{\partial T_1}{\partial x} + \lambda_2 \frac{\partial T_2}{\partial x}, \ T_1 = T_2 = T_0, \ x = X(t), \tag{3}$$

where T_1 is the temperature in the melt and T_2 is that in the crystal, while condition (3) reflects the heat balance at the crystallization front.

The self-modeling solution to (1)-(3) takes the form

$$X(t) = (X_0 + 4\alpha^2 a_1 t)^{1/2}, \ k^2 = \frac{a_1}{a_2},$$
$$T_1 = T' - \frac{T' - T_0}{\operatorname{erfc} \alpha} \operatorname{erfc}\left(\frac{\alpha x}{X}\right), \tag{4}$$

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$$T_2 = T'' - \frac{(T_0 - T'')(1+\operatorname{erf}(k\alpha x/X))}{1+\operatorname{erf}(k\alpha)},$$

where the dimensionless coefficient α is defined from (3). The following relations are obeyed at the crystallization front [x = X(t)]:

$$\frac{\partial T_1}{\partial x} = \frac{T' - T_0}{\operatorname{erfc}\alpha}\frac{2\alpha\exp(-\alpha^2)}{\sqrt{\pi}X}, \tag{5}$$

$$\frac{\partial T_2}{\partial x} = \frac{T_0 - T''}{(1+\operatorname{erf}(k\alpha))}\frac{2k\alpha\exp(-k^2\alpha^2)}{\sqrt{\pi}X},$$

$$\frac{\partial^2 T_1}{\partial x^2} = -\frac{2\alpha^2}{X}\frac{\partial T_1}{\partial x}, \quad \frac{\partial^2 T_2}{\partial x^2} = -\frac{2k^2\alpha^2}{X}\frac{\partial T_2}{\partial x}. \tag{6}$$

For small perturbations superimposed on the phase-transition surface, the problem takes the form

$$\frac{\partial T_i'}{\partial t} = a_i \Delta T_i', \; T_i' + \frac{\partial T_i}{\partial x}E = -T_0 K\Gamma, \; i = 1, 2,$$
$$\frac{\partial E}{\partial t} = -\lambda_1\left(\frac{\partial T_1'}{\partial x} + \frac{\partial^2 T_1}{\partial x^2}E\right) + \lambda_2\left(\frac{\partial T_2'}{\partial x} + \frac{\partial^2 T_2}{\partial x^2}E\right), \tag{7}$$

where T_i' are the perturbations in the temperature distribution; E, small perturbation in the shape of the crystallization front; K(E) is positive if the surface is concave toward the crystal; and Γ, effective surface tension. We introduce the dimensionless variables

$$\varphi_i = \frac{T_i'}{T_0}, \; \tau = \frac{u^2}{a_1}t, \; \xi = \frac{u}{a_1}[x - X(t)], \tag{8}$$
$$\begin{Bmatrix}\eta\\ \zeta\end{Bmatrix} = \frac{u}{a_1}\begin{Bmatrix}y\\ z\end{Bmatrix}, \; \varepsilon = \frac{u}{a_1}E, \; u = \frac{dX}{dt} = \frac{2\alpha^2 a_1}{X},$$

to get the following system of equations:

$$\chi_i\varepsilon + \varphi_i = -k_1\gamma, \; \xi = 0, \; i = 1, 2, \; \gamma = \frac{u\Gamma}{a_1}, \tag{9}$$
$$\frac{\partial\varepsilon}{\partial\tau} = -\nu_1\left(\frac{\partial\varphi_1}{\partial\xi} + L_1\varepsilon\right) + \nu_2\left(\frac{\partial\varphi_2}{\partial\xi} + L_2\varepsilon\right),$$

where

$$\chi_i = \left.\frac{\partial T_i}{\partial x}\right|_{x=X(t)}\frac{X(t)}{2\alpha^2 T_0}; \; \nu_i = \frac{\lambda_i T_0}{a_1};$$
$$L_1 = -\frac{(T' - T_0)\exp(-\alpha^2)}{\sqrt{\pi}\operatorname{erfc}\alpha\,\alpha T_0};$$
$$L_2 = -\frac{(T_0 - T'')k^3\exp(-k^2\alpha^2)}{\sqrt{\pi}(1+\operatorname{erf}(k\alpha))\alpha T_0};$$

$k_1(\varepsilon)$ is the dimensionless curvature of the crystallization surface in the variables of (8).

We represent the functions φ_1, φ_2 in the form

$$\begin{Bmatrix}\varphi_i\\ \varepsilon\end{Bmatrix} = \begin{Bmatrix}\Phi_i(\xi)\\ E_0\end{Bmatrix}\exp\{\omega\tau + i(\varkappa_\eta\eta + \varkappa_\zeta\zeta)\}, \; i = 1, 2,$$

to get $\Phi_i(\xi) = \Phi_i\exp(-\alpha_i\xi)$, where $\varkappa^2 = \varkappa_\eta^2 + \varkappa_\zeta^2$; $\alpha_i = (1/2)(\beta_i + (\beta_i^2 + 4(\beta_i\omega + \varkappa^2))^{1/2})$; $\beta_1 = 1$, $\beta_2 = k^2$.

From conditions (9) we have a system of three homogeneous linear equations in the constants Φ_1, Φ_2, and E_0. The characteristic equation for this system defines ω as a function of the dimensionless parameter $\varkappa$ and the parameter γ:

$$\omega = -\nu_2 L_2\left(\frac{\nu_1 L_1}{\nu_2 L_2} - 1\right) - \nu_2\alpha_2\chi_2\left(\frac{\nu_1\alpha_1\chi_1}{\nu_2\alpha_2\chi_2} - 1\right) - \nu_2\alpha_2\left(\frac{\nu_1\alpha_1}{\nu_2\alpha_2} - 1\right)\gamma\varkappa^2 = \mu_1(1 - \gamma_1\varkappa^2). \qquad (10)$$

If the melt is supercooled, i.e., $T' < T_0$, then it is readily seen that $\mu_1 > 0$. We use the characteristic values for the physical constants for a one-component melt to get that in this case $\gamma_1 > 0$ [8]. Therefore, perturbations with the following wave numbers are unstable:

$$\varkappa < \frac{1}{\sqrt{\gamma_1}}. \qquad (11)$$

We get the following formula for the dimensional perturbation wavelength λ:

$$\lambda > 2\pi\left(\frac{\Gamma a_1}{u}\right)^{1/2}.$$

From (10) we can write an equation to determine ω as a function of γ and $\varkappa$, which is to be solved numerically. As a result we get that an idealized unbounded planar front is unstable in relation to sufficiently long-wave perturbations in the case of a supercooled melt, because such perturbations are acted on by the weak surface tension. If the melt is superheated ($\Delta T_1 > 0$), then on the basis of (3) we readily get that $\omega > 0$ and a planar front is stable. This agrees with physical concepts on the process.

If the crystallization front is spherical or cylindrical, the basic relations derived in [1] are retained, since the temperature distribution within the crystal is homogeneous. These relations differ from those in the present situation in that parameter γ has a different dependence on the parameters. The conclusions of [1] are readily transferred to this case.

In [5], the following formula was derived for the displacement of the interface:

$$\zeta(\rho, t) \sim B_0\int_0^\infty J_0(k\rho)\exp(\varphi(k, t))\,kdk,$$

where $\varphi(k, t) = k(2\alpha\sqrt{Dt} - \alpha_1 kDt)$; here k, D, and t are the dimensional wave number, the diffusion coefficient, and time, correspondingly. The method of steepest descent was used to calculate the integral, and it was found that for large t the value of $\zeta(\rho, t)$ is proportional to $\exp(\beta t^{1/4})$. By virtue of the fact that the characteristic transverse dimension of the crystallization front is bounded in a real system, there is stable motion of a planar front with a speed greater than $u_L = 2\pi D/\varkappa_L L$, where $\varkappa_L$ is the minimum permissible wave number. Instability sets in when the speed of the crystallization front becomes less than u_L. The instant when u_L is attained is $t_L = 4u_L^2\alpha^2/D$. We pass to dimensionless variables in accordance with the formulas $\tau = u_L^2 t/D$, $\varkappa = kD/u_L$. Then for φ we get $\varphi(k, t) = \varkappa(2\alpha\sqrt{\tau} - \alpha_1\varkappa^2\tau)$, where $\varkappa$ can be taken as small.

The integral can be calculated for large τ by the steepest-descent method. But then $4\alpha^2 \ll \tau$, and consequently $\tau_L \ll \tau$. On the other hand, the perturbations become unstable at times of the order of τ_L, and therefore a planar interface between the phases may not exist at time τ. Consequently, it is undesirable to use the steepest-descent method in this case, and the conclusion that the perturbations increase in proportion to $\exp(\beta t^{1/4})$ is incorrect.

NOTATION

a_1, a_2, melt and crystal thermal diffusivity; K, k_1, curvature; k, melt-to-crystal thermal diffusivity ratio; L_1, L_2, ν_1, ν_2 γ, parameters introduced in (9); T_1, melt temperature; T_2, crystal temperature; u_L, critical velocity; X, front coordinate; x, y, z, linear coordinates; x', planar front perturbation; α, growth-rate parameter; Γ, surface tension; ε, E_0, dimensionless perturbation of the front shape and amplitude; ξ, η, ζ, dimensionless coordinates; $\varkappa$, dimensionless wave number; λ, perturbation wavelength; τ, dimensionless time; Φ_1, Φ_2, dimensionless amplitudes of temperature perturbations; ω, dimensionless perturbation increment. Subscripts: $-\infty$ and $+\infty$, state far from the front; 0, state at the front.

LITERATURE CITED

1. Yu. A. Buevich, "Instability in a self-modeling phase-transition front," Inzh.-Fiz. Zh., 40, No. 5, 818-827 (1981).
2. W. Mullins and R. Sekerka, "Stability of a planar phase interface in crystallization of a dilute binary alloy," in: Crystal-Growth Problems [Russian translation], Mir, Moscow (1968), pp. 106-126.
3. J. Cutler and W. Tiller, "Allowance for particle-attachment kinetics to a crystal in analyzing the stability of a cylinder crystallizing from a binary melt," in: Crystal-Growth Problems [Russian translation], Mir, Moscow (1968), pp. 176-196.
4. W. Mullins and R. Sekerka, "The morphological stability of a particle growing by diffusion or heat loss," in: Crystal-Growth Problems [Russian translation], Mir, Moscow (1968), pp. 89-105.
5. Yu. A. Bychkov and S. V. Iordanskii, "Instability of a phase boundary during a phase transformation," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 45-51 (1980).
6. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press (1959).
7. L. I. Rubinshtein, Stefan's Problem [in Russian], Zinatne, Riga (1967).
8. I. K. Kikoin (ed.), Tables of Physical Quantities (Handbook) [in Russian], Atomizdat, Moscow (1976).
9. B. Ya. Lyubov, The Theory of Crystallization in Large Volumes [in Russian], Nauka, Moscow (1975).

THERMOELASTIC STABILITY OF COOLED LASER MIRRORS

V. V. Kharitonov and S. B. Koshelev

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The article examines the interconnection between permissible thermal stresses, deformations, and thermal loads on laser mirrors fixed by different systems.

Installations based on powerful lasers contain on the path of the light a large number of elements, especially mirrors. On each of them the incident wave front is being distorted. Accumulation of distortions on different elements leads to defocusing of the beam and makes it unsuitable for practical purposes. The quality of the mirrors, as one of the causes of distortion of the wave front in multielement systems, must therefore satisfy particularly stringent requirements. Normal deformations must not exceed 1/10-1/40 of the wavelength of the laser beam [1]. In addition, the transverse temperature gradient in the mirror, which is proportional to the absorbed thermal flux, may cause impermissible stresses in it. Thus the permissible luminous loads on laser mirrors are limited by the permissible thermal strains and stresses. The present article shows how the condition of mounting a plane mirror and the intensity of cooling affect the thermal stresses and strains in the mirror and the permissible luminous load imposed on the mirror.

The simplest form of a laser mirror is a plane disk with constant thickness δ and radius R. One surface of the disk is illuminated (heated), and the other surface is cooled by a heat carrier with constant heat transfer coefficient α. The intensity of the irradiation is uniform over its entire surface, i.e., it does not depend on the radial coordinate. (The case with nonuniform illumination requires a special analysis.)

We will first examine two limit cases of mounting mirrors: freely supported by a rigid base and rigidly secured on its circumference.

When a mirror is heated by a laser pulse, the pulse duration t is such that the thickness $\sqrt{at}$ of the heated layer (within the time that the pulse acts) is much smaller than the thickness δ of the mirror; the temperature field in it T(x, t) is correlated with the pulse energy I (J) by the equation of thermal balance

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